Review for Exam 2

This exam covers sections 6.2, 6.3, 6.4, 7.2, 7.3, and 7.4 of the text.

It will include a question asking you to prove one or more of the following formulas: (i) $\frac{d}{dx}e^x = e^x$; (ii) $\frac{d}{dx}\ln x = \frac{1}{x}$; (iii) $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$; or (iv) $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.

The proof of formula (i) goes like this:

$$\begin{aligned}\frac{d}{dx}e^x &= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}\\ &= \lim_{h\to 0}\frac{e^x e^h - e^x}{h}\\ &= \lim_{h\to 0} e^x\left(\frac{e^h-1}{h}\right)\\ &= e^x \lim_{h\to 0}\frac{e^h-1}{h}\\ &= e^x\cdot 1 = e^x.\end{aligned}$$

The proofs of formulas (ii), (iii), and (iv) are all similar. You can find all three proofs in your lecture notes, or slightly longer versions in the text on pages 411-412, 456, and 458. Either version is okay for the test.

The remaining problems on the test will be similar to the homework problems from assignments 4 through 7. You should also review the quiz problems from quizzes 3 and 4.

Here is a section-by-section guide to the material in the text that will be covered on this test.

6.2 This section introduces the formula for "volume by slices": $V = \int_a^b A(x)\,dx$. You should review the entire section, except that you don't need to read Examples 7, 8, and 9, since I won't be asking questions like these examples on the exam. However, it would be a good idea to read one of these examples anyway; the increased understanding you'd gain would be worth it.

To understand what the formula for volume by slices means, you have to visualize a picture like that in Figure 3 on page 355. The object whose volume you are computing is being cut by a slicer, and the slices are perpendicular to the x-axis. The area of the cross-section of the solid obtained by slicing at location x is $A(x)$.

If the object is a solid of revolution around the x-axis, then the cross-sections are either discs, as in Figure 6, or "washers" (discs with holes in them), as in Figure 8. If the object is not a solid of revolution around the x-axis, then the cross-sections will not be circles: they could be squares as in Example 8 or triangles as in Examples 7 and 9. But as I mentioned above, I won't ask questions on the test about such solids; the only solids you will see on the test will be solids of revolution.

You should remember that if the slices are perpendicular to the y-axis, then the volume by slices formula becomes $V = \int_c^d A(y)\,dy$ instead. In general, to make sure you are integrating with respect to the correct variable in your volume formula, you should visualize the "elements of volume" you are using — the small pieces you are putting together to make the entire volume — and take the variable of integration to measure the "thin" direction of the element. If the elements are thin discs perpendicular to the x-axis, the thin direction will be the x-direction.

6.3 There is another formula for volumes of solids of revolution which is quite different in origin from the formula for volume by slices. This is the formula for "volume by cylindrical shells". To review these, re-read section 6.3 in its entirety. Notice that in all the figures in this section, the shells are thin in the x-direction, so the figures correspond to integrals with respect to x. In example 3, there is an integral with respect to y, but the shells are not drawn in the accompanying figure. Can you visualize what they would look like?

6.4 There are a variety of different problems about the physical notion of "work" in this section, but the only type of problem I might ask is the type about computing the work required to pump water out of a tank, as in Example 5. You should read from the beginning of the section through the end of Example 1; then you can skip straight to Example 5 and read that.

7.2 The first part of this section is review from pre-calculus, but you should go over it again anyway. Especially you should make sure that the laws of exponents (in the box labeled "Theorem 2" at the bottom of page 394) are second nature to you, so you don't have to waste time and effort thinking about them on the exam.

You can skip the section titled "Applications of Exponential Functions" on pp. 395–6. I might mention, though, that there's been a pretty good "math for the masses" column running in the New York Times these past few weeks, and this weeks had a little bit about exponential functions and logarithms — if you want to take a quick break from studying you might check it out at

`http://opinionator.blogs.nytimes.com/2010/03/28/power-tools`

(Of course, any quick break on the internet runs the risk of turning into an extended break.)

You can also skip the material on pages 396 and 397, including the "Definition of e" on page 397. This material is designed to lead into the formula for the derivative of e, but I gave a different proof of this formula in class (the one reproduced at the beginning of this review sheet). I also gave a different definition of e: namely, that

$$e = \lim h \to 0(1+h)^{1/h}.$$

You should, of course, know the formula for the derivative of e^x on page 398 (and its proof, given at the beginning of this review sheet). You should also review Examples 2 and 5, and all the material on pages 400 and 401.

7.3 Section 7.3 is actually review from precalculus, but in my experience a lot of calculus students need to review this material. In particular, the laws of logarithms in the box at the bottom of p. 405 should be second nature, just like the laws of exponents. Also, remember that if a property of logarithms ISN'T in this box, then it probably isn't true. For example, there is NO law of logarithms concerning the expressions $\ln(x + y)$ or $(\ln x)^y$.

7.4 In this course we cover all of section 7.4, but for the second exam you won't need to know the material in the last two subsections, titled "General Logarithmic and Exponential Functions" and "Logarithmic Differentiation". That material won't be covered till the third exam. So for the second exam, you can just review pages 411 to 415.