Math 3333, Quiz VII, March 25, 2008

1. Determine whether $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ belong to the span of $\{A_1, A_2, A_3\}$ where

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}.$$

We want to see if we can find real numbers a_1, a_2, a_3 such that $A = a_1A_1 + a_2A_2 + a_3A_3$. This gives us

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 + 2a_3 & -a_1 + a_2 + 2a_3 \\ -a_3 & 3a_1 + 2a_2 + a_3 \end{bmatrix} \Rightarrow \begin{array}{c} a_1 + a_2 + 2a_3 = 1 \\ -a_1 + a_2 + 2a_3 = 0 \\ -a_3 = 2 \\ 3a_1 + 2a_2 + a_3 = 1 \end{array}$$

The augmented matrix for the linear system is

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 2 \\ 3 & 2 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1/2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -15/2 \end{array}\right)$$

The last matrix is obtained by applying suitable elementary row operations. From the last row, it is clear that the linear system is inconsistent. Hence. A does not belong to the span of $\{A_1, A_2, A_3\}$.

2. Determine whether the set of vectors

$$S = \{\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 1 & 0 \end{pmatrix}\}$$

span R_4.

Let $\mathbf{v} = \begin{pmatrix} a & b & c & d \end{pmatrix}$ be any vector in R_4. We have to see if we can find real numbers a_1, a_2, a_3, a_4 such that $\mathbf{v} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 + a_4\mathbf{v}_4$. This give us the linear system

$$\begin{array}{c} a_1 + a_3 + a_4 = a \\ a_2 + a_3 + a_4 = b \\ a_3 + a_4 = c \\ a_1 + a_3 = d \end{array}$$

The augmented matrix and the row echelon form is given by

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 1 & 0 & 1 & 0 & d \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & a \\ 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & c \\ 0 & 0 & 0 & 1 & a-d \end{array}\right)$$

This gives us the linear system

$$\begin{array}{c} a_1 + a_3 + a_4 = a \\ a_2 + a_3 + a_4 = b \\ a_3 + a_4 = c \\ a_4 = a - d \end{array} \quad \Rightarrow \quad \begin{array}{c} a_1 = a - c \\ a_2 = b - c \\ a_3 = c - a + d \\ a_4 = a - d \end{array}$$

Hence, $\mathrm{Span}(S) = R_4$.