Math 3333, Quiz V, March 4, 2008

1. Let

$$A = \begin{bmatrix} 1 & 7 & 0 & 3 \\ -1 & 0 & 2 & 5 \\ 2 & 3 & -4 & 0 \\ 0 & -3 & -1 & 2 \end{bmatrix}.$$

(a) Find the cofactors A_{23} and A_{42}.

- $A_{23} = (-1)^{2+3}\det(\begin{bmatrix} 1 & 7 & 3 \\ 2 & 3 & 0 \\ 0 & -3 & 2 \end{bmatrix}) = 40.$
- $A_{42} = (-1)^{4+2}\det(\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 5 \\ 2 & -4 & 0 \end{bmatrix}) = 20.$

(b) Suppose we have $A_{11} = -87, A_{12} = 10, A_{13} = -36, A_{14} = -3, A_{21} = 101, A_{22} = -14, A_{24} = -1, A_{31} = 81, A_{32} = -12, A_{33} = 38, A_{34} = 1, A_{41} = -122, A_{43} = -46, A_{44} = -6$. Then find the adjoint matrix $\text{adj}(A)$ and A^{-1}, if it exists.

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{bmatrix} = \begin{bmatrix} -87 & 101 & 81 & -122 \\ 10 & -14 & -12 & 20 \\ -36 & 40 & 38 & -46 \\ -3 & -1 & 1 & -6 \end{bmatrix}$$

We have $\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} + a_{14}A_{14} = (1)(-87) + (7)(10) + (0)(-36) + (3)(-3) = -26$. Since $A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$, we get

$$A^{-1} = \begin{bmatrix} 87/26 & -101/26 & -81/26 & 122/26 \\ -10/26 & 14/26 & 12/26 & -20/26 \\ 36/26 & -40/26 & -38/26 & 46/26 \\ 3/26 & 1/26 & -1/26 & 6/26 \end{bmatrix}$$

2. Solve the following linear system **using Cramer's rule**.

$$2x + 3y = 5, \qquad x - 5y = 2$$

The matrix form of the linear system is $A\mathbf{x} = \mathbf{b}$, with $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$. Replacing the first column of A by $\mathbf{b}$ we get the matrix $A_1 = \begin{bmatrix} 5 & 3 \\ 2 & -5 \end{bmatrix}$ and replacing the second column of A by $\mathbf{b}$ we get the matrix $A_2 = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$. Now Cramer's rule tells us that

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-31}{-13} = \frac{31}{13} \text{ and } x_2 = \frac{\det(A_2)}{\det(A)} = \frac{-1}{-13} = \frac{1}{13}.$$