Math 3333, Quiz IV, February 12, 2008

1. (10 Points) Determine all the solutions of the following linear system by obtaining a row echelon form of its augmented matrix.

$$x + 2y - z = 2, \qquad 2x + z = 4, \qquad 4x + 4y - z = -1.$$

The augmented matrix of the above linear system is $\left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 0 & 1 & 4 \\ 4 & 4 & -1 & -1 \end{array}\right)$. We apply elementary row operations to obtain a row echelon form matrix.

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 0 & 1 & 4 \\ 4 & 4 & -1 & -1 \end{array}\right) \xrightarrow[-4\mathbf{r}_1+\mathbf{r}_3\to\mathbf{r}_3]{-2\mathbf{r}_1+\mathbf{r}_2\to\mathbf{r}_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 3 & 0 \\ 0 & -4 & 3 & -9 \end{array}\right) - \mathbf{r}_2 + \mathbf{r}_3 \to \mathbf{r}_3 \left(\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 3 & 0 \\ 0 & 0 & 0 & -9 \end{array}\right).$$

The last row of the matrix tells us that we have to satisfy $0 = -9$, which is impossible. Hence, the linear system has no solutions.

2. (10 Points) Determine which of the following matrices are in reduced row echelon form (RREF), row echelon form (REF) or neither (N).

(i) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$ $\boxed{\text{N}}$ (ii) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$ $\boxed{\text{REF}}$ (iii) $\begin{pmatrix} 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\boxed{\text{RREF}}$

(iv) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\boxed{\text{RREF}}$ (v) $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\boxed{\text{REF}}$