State whether the following statements are true or false.

1. The following linear system is consistent.

$$x + z = 2, \qquad x - y + 2z = 0, \qquad 3x + 2y + z = 5.$$

We have $2 \times$ (eqn. two) + (eqn. three) : $5x + 5z = 5 \Rightarrow x + z = 1$. But we cannot satisfy both $x + z = 1$ and $x + z = 2$. Hence, the system has no solution, which means the system is inconsistent. Hence the statement is $\boxed{\text{FALSE}}$

2. We have

$$\begin{pmatrix} 1 & 0 \\ -3 & 2 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} 4 & 0 & 2 \\ 1 & -3 & 11 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 5 & 3 \end{pmatrix}$$

If we multiply a 3×2 matrix with a 2×3 matrix, we get a 3×3 matrix, not a 2×2 matrix. Hence, the statement is $\boxed{\text{FALSE}}$

3. Let

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 5 & 7 \\ -1 & -3 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 & 9 \\ 0 & 5 & 7 \end{pmatrix}.$$

Then

$$(A^T B)^T = C.$$

We have $A^T = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 5 & -3 \\ 1 & 7 & 1 \end{pmatrix}$. Hence $A^T B = \begin{pmatrix} 1 & 0 \\ 2 & 5 \\ 9 & 7 \end{pmatrix}$. This implies that

$$(A^T B)^T = \begin{pmatrix} 1 & 2 & 9 \\ 0 & 5 & 7 \end{pmatrix} = C.$$

Hence the statement is $\boxed{\text{TRUE}}$