Solution to HW 4

Section 2.2

⑦ (a) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ \boxed{1} & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{-r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{r_2\leftrightarrow r_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 3 \end{array}\right]$

$\xrightarrow{-r_3\to r_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \Rightarrow \begin{array}{r} x+y+z=0 \\ y+z=1 \\ z=-3 \end{array} \Rightarrow \boxed{x=-1,\ y=4,\ z=-3.}$

⑦ (b) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ \boxed{1} & 1 & 1 & 0 \\ \boxed{1} & 1 & 2 & 0 \end{array}\right] \xrightarrow[-r_1+r_3\to r_3]{-r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right] \xrightarrow{-r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & \boxed{-1} & -1 & 0 \end{array}\right]$

$\xrightarrow{r_2+r_3\to r_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{array}{r} x+2y+3z=0 \\ y+2z=0 \\ z=0 \end{array} \Rightarrow \boxed{x=0,\ y=0,\ z=0}$

⑦ (c) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ \boxed{1} & 1 & 1 & 0 \\ \boxed{5} & 7 & 9 & 0 \end{array}\right] \xrightarrow[-5r_1+r_3\to r_3]{-r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \xrightarrow{-r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & \boxed{-3} & -6 & 0 \end{array}\right]$

$\xrightarrow{3r_2+r_3\to r_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{r} x+2y+3z=0 \\ y+2z=0 \end{array}$ let $z=r$ any real no. then $y=-2r$ & $x=r$

Hence sol^n: $\boxed{x=r,\ y=-2r,\ z=r \text{ for any real no. } r.}$

⑦ (d) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ \boxed{1} & 2 & 1 & 0 \end{array}\right] \xrightarrow{-r_1+r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}r_2\to r_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\Rightarrow \begin{array}{r} x+2y+3z=0 \\ z=0 \end{array} \Rightarrow z=0,\ x+2y=0$ let $y=$ any real no. r then $x=-2r$

Hence sol^n is $\boxed{x=-2r,\ y=r,\ z=0 \text{ for any real no. } r.}$

(9) (b)
$$\left[\begin{array}{ccc|c}1&2&1&0\\ \boxed{2}&3&0&0\\ 0&1&2&0\\ \boxed{2}&1&4&0\end{array}\right] \xrightarrow[-2r_1+r_4\to r_4]{-2r_1+r_2\to r_2} \left[\begin{array}{ccc|c}1&2&1&0\\ 0&-1&-2&0\\ 0&1&2&0\\ 0&-3&2&0\end{array}\right] \xrightarrow{-r_2\to r_2} \left[\begin{array}{ccc|c}1&2&1&0\\ 0&1&2&0\\ 0&\boxed{1}&2&0\\ 0&\boxed{-3}&2&0\end{array}\right]$$

$$\xrightarrow[3r_2+r_4\to r_4]{-r_2+r_3\to r_3} \left[\begin{array}{ccc|c}1&2&1&0\\ 0&1&2&0\\ 0&0&0&0\\ 0&0&8&0\end{array}\right] \xrightarrow{r_3\leftrightarrow r_4} \left[\begin{array}{ccc|c}1&2&1&0\\ 0&1&2&0\\ 0&0&8&0\\ 0&0&0&0\end{array}\right] \xrightarrow{\frac{1}{8}r_3\to r_3} \left[\begin{array}{ccc|c}1&2&1&0\\ 0&1&2&0\\ 0&0&1&0\\ 0&0&0&0\end{array}\right]$$

$$\Rightarrow \left.\begin{aligned} x+2y+z&=0\\ y+2z&=0\\ z&=0\end{aligned}\right\} \Rightarrow \boxed{x=0,\ y=0,\ z=0}$$

(12) $A\underline{x}=3\underline{x} \Rightarrow (A-3I_3)\underline{x}=\underline{0}$

$$A-3I_3=\begin{bmatrix}1&2&-1\\1&0&1\\4&-4&5\end{bmatrix}-\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}=\begin{bmatrix}-2&2&-1\\1&-3&1\\4&-4&2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-2&2&-1\\1&-3&1\\4&-4&2\end{bmatrix}\underline{x}=\underline{0}$$

Augmented matrix:
$$\left[\begin{array}{ccc|c}-2&2&-1&0\\ 1&-3&1&0\\ 4&-4&2&0\end{array}\right] \xrightarrow{r_1\leftrightarrow r_2} \left[\begin{array}{ccc|c}1&-3&1&0\\ \boxed{-2}&2&-1&0\\ \boxed{4}&-4&2&0\end{array}\right]$$

$$\xrightarrow[4r_1+r_3\to r_3]{r_1+r_2\to r_2} \left[\begin{array}{ccc|c}1&-3&1&0\\ 0&-4&1&0\\ 0&8&-2&0\end{array}\right] \xrightarrow{-\frac{1}{4}r_2\to r_2} \left[\begin{array}{ccc|c}1&-3&1&0\\ 0&1&-1/4&0\\ 0&\boxed{8}&-2&0\end{array}\right] \xrightarrow{-8r_2+r_3\to r_3} \left[\begin{array}{ccc|c}1&-3&1&0\\ 0&1&-1/4&0\\ 0&0&0&0\end{array}\right]$$

$$\Rightarrow \begin{aligned} x_1-3x_2+x_3&=0\\ x_2-\tfrac{1}{4}x_3&=0\end{aligned}$$

Let $x_3=r$, any real no. then $x_2=\frac{1}{4}r$ & $x_1=-\frac{1}{4}r$

So, the solⁿ is $\begin{bmatrix}-r/4\\ r/4\\ r\end{bmatrix}$, for any real no. r.

(15) Augmented matrix:
$$\left[\begin{array}{ccc|c}1&1&1&2\\ \boxed{2}&3&2&5\\ \boxed{2}&3&a^2-1&a+1\end{array}\right] \xrightarrow[-2r_1+r_3\to r_3]{-2r_1+r_2\to r_2} \left[\begin{array}{ccc|c}1&1&1&2\\ 0&1&0&1\\ 0&\boxed{1}&a^2-3&a-3\end{array}\right]$$

$$\xrightarrow{-r_2+r_3\to r_3} \left[\begin{array}{ccc|c}1&1&1&2\\ 0&1&0&1\\ 0&0&a^2-3&a-4\end{array}\right]=M$$

Case (i) $a = \sqrt{3}$ $\Rightarrow$ $M = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & \sqrt{3}-4 \end{array}\right]$ $\Rightarrow$ no solutions

non-zero

Case (ii) $a = -\sqrt{3}$ $\Rightarrow$ $M = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -\sqrt{3}-4 \end{array}\right]$ $\Rightarrow$ no solution

non-zero

Case (iii) $a \neq \pm\sqrt{3}$ i.e. $a^2-3 \neq 0$ $\quad -\frac{1}{a^2-3} r_3 \to r_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{a-4}{a^2-3} \end{array}\right]$

$$\Rightarrow \left.\begin{array}{r} x+y+z=2 \\ y=1 \\ z=\frac{a-4}{a^2-3} \end{array}\right\} \Rightarrow \quad x = 1 - \frac{a-4}{a^2-3} = \frac{a^2-a+1}{a-3}$$

$$y = 1$$

$$z = \frac{a-4}{a^2-3}$$

<u>Conclusion</u>: No solution for $a = \pm\sqrt{3}$

Unique solution for $a \neq \pm\sqrt{3}$

infinitely many solutions : <u>never</u>.

(22) $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ augmented matrix: $\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right]$

$r_1 \leftrightarrow r_3$ $\left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right]$ $\quad \frac{r_1}{2} \to r_1$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ \boxed{2} & -1 & 3 & b \\ \boxed{4} & 1 & 3 & a \end{array}\right]$ $\begin{array}{l} -2r_1 + r_2 \to r_2 \\ -4r_1 + r_3 \to r_3 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & \boxed{-3} & 3 & a-2c \end{array}\right]$

$-r_2 + r_3 \to r_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & 0 & 0 & a-b-c \end{array}\right]$ For system to have solution, we must have $\boxed{a-b-c=0}$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & 0 & 0 & 0 \end{array}\right]$ sol^n is $z = r$, any real no.

$y = r - \frac{b-c}{3}$, $x = \frac{b}{3} + \frac{c}{6} - r$.

Section 2.3

9 (a) $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$: $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ \boxed{2} & 6 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1 + r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$

$\Rightarrow$ A is row equivalent to $\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$, which has a zero row $\Rightarrow$ A is singular.

9 (b) $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$: $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ \boxed{-2} & 6 & 0 & 1 \end{array}\right] \xrightarrow{2r_1 + r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 12 & 2 & 1 \end{array}\right] \xrightarrow{\frac{1}{12} r_2 \to r_2}$

$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1/6 & 1/12 \end{array}\right] \xrightarrow{-3r_2 + r_1 \to r_1} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/4 \\ 0 & 1 & 1/6 & 1/12 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$

9 (c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$: $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ \boxed{1} & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1 + r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{-r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_2 + r_3 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 2 & \boxed{3} & 1 & 0 & 0 \\ 0 & 1 & \boxed{1} & 1 & -1 & 0 \\ 0 & 0 & +1 & -1 & 1 & 1 \end{array}\right]$ $\begin{array}{l} -r_3 + r_2 \to r_2 \\ -3r_3 + r_1 \to r_1 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & \boxed{2} & 0 & 4 & -3 & -3 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{-2r_2 + r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

9 (d) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$: $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ \boxed{1} & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1 + r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$

$\xrightarrow{-r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & \boxed{1} & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_2 + r_2 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$

$\Rightarrow$ A is row equivalent to $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, which has a zero row $\Rightarrow$ A is singular

11 (a) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ \boxed{1} & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1 + r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_2 + r_3 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right]$

$\xrightarrow{-r_3 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & \boxed{1} & 1 & 0 & 0 \\ 0 & 1 & \boxed{2} & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \xrightarrow[-2r_3 + r_2 \to r_2]{-r_3 + r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & \boxed{1} & 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \xrightarrow{-r_2 + r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

11 (b)

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ \boxed{1} & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ \boxed{1} & -1 & 2 & 1 & 0 & 0 & 1 & 0 \\ \boxed{1} & 3 & 3 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \\ -r_1+r_4 \to r_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & \boxed{-2} & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & \boxed{2} & 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} 2r_2+r_3 \to r_3 \\ -2r_2+r_3 \to r_3 \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 2 & -3 & 2 & 1 & 0 \\ 0 & 0 & 6 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \quad 2r_2+r_3 \to r_3 \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -3 & 2 & -3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & -5 & 2 & 2 & 1 \end{array}\right]$$

$\Rightarrow A$ is row equivalent to $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, which has a zero row $\Rightarrow$ $\boxed{A \text{ is singular}}$

11 (c)

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 5 & 9 & 1 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} 2r_1+r_2 \to r_2 \\ -2r_3+r_4 \to r_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 5 & 3 & 4 & 2 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 5 & 3 & 4 & 0 & 0 & -2 & 1 \end{array}\right]$$

$$-r_2+r_4 \to r_4 \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 5 & 3 & 4 & 2 & 1 & 0 & 0 \\ 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2 & -1 & -2 & 1 \end{array}\right]$$

$\Rightarrow A$ is row equivalent to $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 5 & 3 & 4 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, which has a zero row, $\Rightarrow$ $\boxed{A \text{ is singular}}$

11 (d)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ \boxed{1} & 3 & 2 & 0 & 1 & 0 \\ \boxed{1} & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & \boxed{-2} & 0 & -1 & 0 & 1 \end{array}\right] \; 2r_2+r_3 \to r_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 1 \end{array}\right]$$

$$\tfrac{1}{2}r_3 \to r_3 \left[\begin{array}{ccc|ccc} 1 & 2 & \boxed{1} & 1 & 0 & 0 \\ 0 & 1 & \boxed{1} & -1 & 1 & 0 \\ 0 & 0 & 1 & -3/2 & 1 & 1/2 \end{array}\right] \begin{array}{l} -r_3+r_1 \to r_1 \\ -r_3+r_2 \to r_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & \boxed{2} & 0 & 5/2 & -1 & -1/2 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -3/2 & 1 & 1/2 \end{array}\right] \; -2r_2+r_1 \to r_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -1 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -3/2 & 1 & 1/2 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -3/2 & 1 & 1/2 \end{bmatrix}$$

11 (e)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ \boxed{1} & 3 & 1 & 0 & 1 & 0 \\ \boxed{1} & 1 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & \boxed{-1} & 1 & -1 & 0 & 1 \end{array}\right] \; r_2+r_3 \to r_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

$\Rightarrow A$ is row equivalent to $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$, which has a zero row $\Rightarrow$ A is singular

(16) We have $(A^{-1})^{-1} = A$, Hence to find A :

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ \boxed{1} & 1 & 2 & 0 & 1 & 0 \\ \boxed{1} & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2} r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & \boxed{1} & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow{-r_3+r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & \boxed{1} & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow{-r_2+r_1 \to r_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -1 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \Rightarrow A = \begin{bmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1 & 1 & 0 \end{bmatrix}$$