Solutions to HW 3

Section 2.1

(2)(a) $A = \begin{bmatrix} -1 & 1 & -1 & 0 & 3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix} \xrightarrow{-r_1 \to r_1} \begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix}$

$\xrightarrow[4r_1+r_3 \to r_3]{3r_1+r_2 \to r_2} \begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & -2 & -8 & -2 & -2 \end{bmatrix} \xrightarrow{2r_2+r_3 \to r_3} \begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Row echelon form matrix

(5)(a) $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 9 \\ 3 & 2 & 4 \end{bmatrix} \xrightarrow[-3r_1+r_3 \to r_3]{2r_1+r_2 \to r_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \end{bmatrix} \xrightarrow{-2r_2+r_3 \to r_3} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

Row echelon form

(6)(b) $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{bmatrix} \xrightarrow{\substack{-3r_1+r_2 \to r_2 \\ -5r_1+r_3 \to r_3 \\ 2r_1+r_4 \to r_4}} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-r_2+r_3 \to r_3} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{-\frac{1}{2}r_3 \to r_3} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Row echelon form

(7) (a) REF if $w=1$ N if $w \neq 1$ (b) REF (c) RREF

(8) (a) REF (b) RREF (c) N

(13) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (i) Suppose $\theta \neq \pi/2, 3\pi/2$ i.e. $\cos\theta \neq 0$

then $A \xrightarrow{\frac{1}{\cos\theta} r_1 \to r_1} = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} \xrightarrow{\tan\theta\, r_1 + r_2 \to r_2} \begin{bmatrix} 1 & \tan\theta \\ 0 & 1+\tan^2\theta \end{bmatrix}$

But $\tan^2\theta + 1 = \sec^2\theta$ $\begin{bmatrix} 1 & \tan\theta \\ 0 & \sec^2\theta \end{bmatrix} \xrightarrow{\frac{1}{\sec^2\theta} r_2 \to r_2} \begin{bmatrix} 1 & \tan\theta \\ 0 & 1 \end{bmatrix}$ row-echelon form

~~I~~ (ii) If $\theta = \pi/2$ $\quad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{-r_1 \to r_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ row echelon form

(iii) If $\theta = 3\pi/2$ $\quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \xrightarrow{-r_2 \to r_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ row echelon form

Section 2.2

5 (a) Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right] \xrightarrow[-3r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & -5 & 6 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3} r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1/3 & 4/3 \\ 0 & -2 & -5 & 6 \end{array}\right] \xrightarrow{2r_2+r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1/3 & 4/3 \\ 0 & 0 & -13/3 & 26/3 \end{array}\right]$$

$$\xrightarrow{-\frac{3}{13} r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1/3 & 4/3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$ row echelon form

Corresponding linear system:

$$\begin{aligned} x + y + 2z &= -1 \\ y + \tfrac{1}{3} z &= \tfrac{4}{3} \\ z &= -2 \end{aligned}$$

Back substitution gives

$$\boxed{z = -2 \;, \; y = 2 \;, \; x = 1}$$

6 (c) Augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$$

To make calculations simpler let us move 3rd row to the top first

$$\xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$$

$-3r_1+r_2 \to r_2$
$-2r_1+r_3 \to r_3$
$-6r_1+r_4 \to r_4$
$-5r_1+r_5 \to r_5$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right] \xrightarrow{-r_4+r_5 \to r_5} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$$

$\Rightarrow \boxed{\text{no solutions}}$