

# Solutions to HW.1

## Section 1.1

$$\textcircled{1} \quad \begin{aligned} x + 2y &= 8 & \longrightarrow & x = 8 - 2y \\ 3x - 4y &= 4 & \longrightarrow & 3(8 - 2y) - 4y = 4 \Rightarrow \boxed{y = 2} \Rightarrow \boxed{x = 4} \end{aligned}$$

$$\textcircled{2} \quad \begin{aligned} 2x - 3y + 4z &= -12 \\ x - 2y + z &= -5 \\ 3x + y + 2z &= 1 \end{aligned} \quad \begin{array}{l} \text{Eliminate } x: \\ y + 2z = -2 \\ 7y - z = 16 \end{array}$$

$$\text{Eliminate } y: \quad -15z = 30 \Rightarrow \boxed{z = -2} \Rightarrow \boxed{y = 2} \Rightarrow \boxed{x = 1}$$

$$\textcircled{13} \quad \begin{aligned} x + 3y &= -4 \\ 2x + 5y &= -8 \\ x + 3y &= -5 \end{aligned} \quad \left. \begin{array}{l} \searrow \\ \nearrow \end{array} \right\} \text{not consistent} \Rightarrow \text{no solution.}$$

## Section 1.2:

$$\textcircled{6} \textcircled{7} \quad 2B + F = 2 \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} \quad \text{cannot add since the 2 matrices have different sizes.}$$

$$\textcircled{7} \textcircled{a} \quad 3D + 2F = 3 \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} + 2 \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & -6 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 10 & 18 \end{bmatrix}$$

$$\textcircled{8} \textcircled{a} \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad (A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\textcircled{8} \textcircled{b} \quad (C+E)^T = \left( \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$C^T + E^T = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}^T + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 & 2 \\ -1 & 1 & 1 \\ 3 & 5 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 3 \\ -4 & 1 & 2 \\ 5 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}$$

$$\textcircled{8} \textcircled{c} \quad 2A^T + B = 2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 6 & 3 \\ 9 & 10 \end{bmatrix}$$

### Section 1.3 :

$$\textcircled{3} \vec{a} \cdot \vec{b} = \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix} = (-3)(-3) + (2)(2) + (x)(x) = 13 + x^2$$

$$\vec{a} \cdot \vec{b} = 17 \Rightarrow 13 + x^2 = 17 \Rightarrow x^2 = 4 \Rightarrow \boxed{x = 2, -2}$$

$$\textcircled{11}(a) AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} (1)(1) + (2)(2) + (3)(3) & (1)(0) + (2)(1) + (3)(2) \\ (2)(1) + (1)(2) + (4)(3) & (2)(0) + (1)(1) + (4)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$\textcircled{11}(b) BA = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} (1)(1) + (0)(2) & (1)(2) + (0)(1) & (1)(3) + (0)(4) \\ (2)(1) + (1)(2) & (2)(2) + (1)(1) & (2)(3) + (1)(4) \\ (3)(1) + (2)(2) & (3)(2) + (2)(1) & (3)(3) + (2)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 10 \\ 7 & 8 & 17 \end{bmatrix}$$

$\textcircled{12}(a)$   $DA + B$  not possible because  $D$  is a  $2 \times 2$  matrix,  $A$  is a  $2 \times 3$  matrix &  $B$  is a  $3 \times 2$  matrix. So cannot add.

$$\textcircled{14}(f) A(C - 3E) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \left( \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3 \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} -3 & 11 & -12 \\ 4 & -2 & -7 \\ -7 & -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -8 & -26 \\ -30 & 0 & -31 \end{bmatrix}$$

$$\textcircled{15}(e) (C + E)^T B = \underbrace{\begin{bmatrix} 5 & 4 & 5 \\ -5 & 2 & 3 \\ 8 & 9 & 4 \end{bmatrix}}_{\text{from sec 1.2 (8)(b)}} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 28 & 14 \\ 8 & 8 \\ -38 & 17 \end{bmatrix} = C^T B + E^T B$$