Math 3333, Quiz VI, November 8, 2007

Name :

1. Find the dimension of the subspace of M_{23} consisting of all matrices of the form $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ with $c = 2a + b, d = b - 3f$ and $e = a - 2b + f$.

$$\begin{aligned}
\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} &= \begin{pmatrix} a & b & 2a+b \\ b-3f & a-2b+f & f \end{pmatrix} \\
&= a\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \end{pmatrix} + f\begin{pmatrix} 0 & 0 & 0 \\ -3 & 1 & 1 \end{pmatrix}
\end{aligned}$$

This implies that the subspace $W = Span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are the three vectors above. To check for linear independence, we have

$$a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = 0 \Rightarrow \begin{pmatrix} a_1 & a_2 & 2a_1 + a_2 \\ a_2 - 3a_3 & a_1 - 2a_2 + a_3 & a_3 \end{pmatrix} = \mathbf{0} \Rightarrow a_1 = a_2 = a_3 = 0$$

Hence, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms a basis for W, which implies that $\dim(W) = 3$.

2. Find the dimension of the subspace of M_{22} consisting of all skew-symmetric matrices.

Any skew-symmetric matrix in M_{22} has the form $\mathbf{v} = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$. Since

$$\mathbf{v} = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} = a\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

we have that $\{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\}$ forms a basis for skew-symmetric matrices in M_{22}. (You do not have to check for linear independence because a set of just one vector is always linearly independent). Hence

$$\dim = 1.$$