Homework 3 : This homework is due on **September** 20.

1. Find the reduced row echelon form of the given matrices. Record the row operations you perform using the notations for elementary row operations. $A = \begin{pmatrix} -1 & 1 & -1 & 0 & 3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & -2 \\ -2 & 1 & 9 \\ 3 & 2 & 4 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{pmatrix}, D = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$

2. Each of the given system is in row-echelon or reduced row-echelon form. Solve the system.
   (a) $x - 3y + 4z + w = 0$, $z - w = 4$, $w = 1$.
   (b) $x + y - z + 2w = 4$, $w = 5$.
   (c) $x + y = 2$, $z + w = -3$.

3. Find all solutions, if any, of the following linear system using Gauss elimination method or Gauss-Jordan elimination method.
   (a) $x + y + 2z + 3w = 13$, $x - 2y + z + w = 8$, $3x + y + z - w = 1$
   (b) $x + 2y + 3z = 0$, $x + y + z = 0$, $x + y + 2z = 0$.
   (c) Linear system with the augmented matrix $\left(\begin{array}{cccc|c} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right)$

4. Find a 3×1 matrix $\mathbf{x}$ with entries not all zero such that
$$A\mathbf{x} = 3\mathbf{x} \text{ where } A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$

5. In the following linear system, determine all the values of a for the which the resulting system has (a) no solution; (b) a unique solution; (c) infinitely many solutions.
$$x + y + z = 2, \;\; 2x + 3y + 2z = 5, \;\; 2x + 3y + (a^2 - 1)z = a + 1.$$

6. Find the inverse of the following matrices, if they exist.
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 5 & 9 & 1 & 6 \end{pmatrix}, D = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ with } ad - bc = 1$$

7. If $A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, find A.

8. For each of the following matrices, find a matrix of the form $\begin{pmatrix} I_r & O_{r,n-r} \\ O_{m-r,r} & O_{m-r,n-r} \end{pmatrix}$:
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & -2 & 3 & 1 \\ 0 & -1 & 4 & 3 \\ 1 & 0 & -2 & -1 \end{pmatrix}, C = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 3 & 2 \\ 3 & 1 & 3 \end{pmatrix}$$