This homework is due on **November** 29.

1. Find a basis for range of $L: R^2 \to R^2$ where $L(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}) = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$. Is $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ in the range ?

2. Let $L: R_2 \to R_3$ where $L(\begin{bmatrix} u_1 & u_2 \end{bmatrix}) = \begin{bmatrix} u_1 & u_1 + u_2 & u_2 \end{bmatrix}$. Is L onto?

3. Find the dimension of range of $L: M_{23} \to M_{33}$ where $L(A) = \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} A$.

4. Find a basis for range of $L: P_2 \to P_1$ where $L(at^2 + bt + c) = (a+b)t + (b-c)$.

5. Find a basis for range of $L: M_{22} \to M_{22}$ where $L(A) = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} A - A \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$.

6. Find a basis for and dimension of the range of $L: R^5 \to R^4$ where $L(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & -1 & 3 & -1 \\ 1 & 0 & 0 & 2 & -1 \\ 2 & 0 & -1 & 5 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$

7. Let $L: R^4 \to R^3$ be defined by $L(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & 1 & 3 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ -1 & -2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$. Let S and T be the natural basis of R^4 and R^3, and consider the ordered bases $S' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ and $T' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ for R^4 and R^3. Find the representation of L w. r. t. $(a) S$ and T; $(b) S'$ and T'.

8. Let $L: R^3 \to R^3$ where $L(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, L(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, L(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Find the representation of L w. r. t. the natural basis of R^3. Find $L(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$ by using the defn of L and also using the matrix.

9. Let $L: P_1 \to P_2$ be defined by $L(p(t)) = tp(t) + p(0)$. Consider the ordered bases $S = \{t, 1\}$ and $S' = \{t+1, t-1\}$ of P_1 and $T = \{t^2, t, 1\}$ and $T' = \{t^2+1, t-1, t+1\}$ of P_2. Find the representation of L with respect to S and T; and S' and T'. Find $L(-3t-3)$ by using the definition of L and the two matrices obtained above.

10. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, and let $L: M_{22} \to M_{22}$ be given by $L(X) = AX - XA$ for X in M_{22}. Let $S = \{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$ and $T = \{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\}$. Find the representation of L with respect to (a) S, (b) T, (c) S and T, (d) T and S.

11. Let $L: R_3 \to R_2$ have the representation w.r.t. natural basis of R_3 and R_2 the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$. Find (a) $L(\begin{bmatrix} 1 & 2 & 3 \end{bmatrix})$, (b) $L(\begin{bmatrix} -1 & 2 & -1 \end{bmatrix})$, (c) $L(\begin{bmatrix} 0 & 1 & 2 \end{bmatrix})$, (d) $L(\begin{bmatrix} 0 & 1 & 0 \end{bmatrix})$ and (e) $L(\begin{bmatrix} 0 & 0 & 1 \end{bmatrix})$.