TALKING MATHEMATICS: AN ABSTRACT ALGEBRA PROFESSOR'S TEACHING DIARIES

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The world of a mathematician, with all its creativity and precision is fascinating to most people. This study is an account of collaboration between mathematicians and mathematics educators. In order to examine a mathematician's daily activities, we have primarily employed Schoenfeld's goal-orientated decision making theory to identify his Resources, Orientations and Goals (ROGs) in teaching an abstract algebra class. Our preliminary results report on a healthy and positive atmosphere where all involved freely express their views on mathematics and pedagogy.

Keywords: Abstract algebra, Mathematician's thought processes, Resources, Orientations, Goals

Literature

Mathematics has been a passion for mathematicians for centuries. Thurston (1994) poses the question: "How do mathematicians advance human understanding of mathematics?" In his view, "what we are doing is finding ways for people to understand and think about mathematics" (p.162). While this statement is encouraging, in reality the communication between the mathematicians and those outside of the community is very limited. According to Byers (2007):

> People want to talk about mathematics but they don't. They don't know how. Perhaps they don't have the language, perhaps there are other reasons. Many mathematicians usually don't talk about mathematics because talking is not their thing-their thing is "doing" of mathematics. Educators talk about teaching mathematics but rarely about mathematics itself. Some educators, like scientists, engineers, and many other professionals who use mathematics don't talk about mathematics because they feel that they don't possess the expertise that would be required to speak intelligently about mathematics (p.7).

Research in teaching and learning at the university level is fairly new. A study by Speer, Smith, and Horvath (2010) showed that research conducted has hardly examined the daily teaching practice of mathematicians and gave the following possible reasons for this lack of research:

> First, lecture can be taken as a description of teaching practice, rather than a common instructional activity within which teaching takes place. Second, the professional culture of mathematics may obscure differences in teaching and forestall discussions of teaching within the set of shared norms. Strong content knowledge and the ability to structure it for students may be taken as sufficient for good teaching. Third, collegiate mathematics teachers have limited exposure to and knowledge of pre-college research, where aspects of practice have been productively analysed (p.111).

On the other hand, some recent research activities are as a result of a partnership between research mathematicians and mathematics educators. In Nardi's (2007) view this is a 'fragile' but never the less a 'crucial' relationship between the two parties. To close this gap, Hodgson (2012), in his plenary lecture at ICME 12, raised the point about the need for a community and forum where mathematicians and mathematics educators can work as closely as possible on teaching and learning mathematics. In recent years various institutes and individuals are more willing to examine and reflect on their own teaching styles and the trend is slowly changing. For example, a study by Paterson, Thomas and Taylor (2011) described a supportive and positive association of two groups of mathematicians and mathematics educators from the same university which allowed the "cross-fertilization of ideas" (*ibid*, p. 359). The group met on a regular basis and discussed teaching strategies while watching small clips of each other's videos during a teaching episode. Similarly, the research conducted by Hannah, Stewart and Thomas (2011) indicated a case in which a mathematician took careful diaries of his actions and thoughts during his linear algebra lectures and reflected on them with two mathematics education researchers.

In describing the role of a mathematician Thurston (1994) clearly declares that: "We are not trying to meet some abstract production quota of definitions, theorems and proofs. The measure of our success is whether what we do enables people to understand and think more clearly and effectively about mathematics" (p. 163). Although, Mason (2002) maintains similar views, he adds that:

> .... This does not mean that it is effective to walk in and solve a lot of problems, formulate definitions and prove theorems in front of them, mindless of their presence. On the other hand, neither is it effective to give a truncated and stylised presentation which supports the impression that mathematics is completely cut, dried and salted away, that it is something that one can either pick up easily or not at all(p. 4).

The aim of this study is to investigate how mathematicians build mathematical knowledge, so in return, help students to think like mathematicians and encourage them to become independent researchers. This study is not about imposing mathematicians what to do or how to teach, rather support and share ideas with one another in order to reflect on the teaching process.

Method

The study described here is a case study that took place at the mathematics department, University of Oklahoma in Fall 2012. Two mathematicians and two mathematics educators formed a community of enquiry to examine how mathematicians think and create mathematical knowledge. The data for this research comes from the research mathematicians' day-to-day reflection of his teaching an abstract algebra course which after each class which he posted it on his website and made it available immediately to the group; the mathematics educator's observation of the classes and her written notes; weekly discussion meetings of the whole group after reading each of these reflections and the recordings of each meeting which was later transcribed for further discussion.

Theoretical framework

We have employed Schoenfeld's (2011) ROGs model to describe the professor's Resources, Orientation and Goals. By resources Schoenfeld focuses mainly on knowledge, which he defines "as the information that he or she has potentially available to bring to bear in order to solve problems, achieve goals, or perform other such tasks" (p. 25). Goals are defined simply as what the individual wants to achieve. The term orientations refer to a group of terms such as "dispositions, beliefs, values, tastes, and preferences" (p. 29). We are also interested in mathematicians' decision making moments as part of this study and would like to examine them in detail.

Preliminary Results

Many of the emerging results deal with the professor's orientations. For example, he decided to cover ring and field theory prior to group theory (and chose the course textbook accordingly). His rationale for this approach included that he himself had been taught rings before groups, and that he believes that fields are more intuitive and familiar to students than groups. Another emerging theme in the data is both the tension and collaboration between his orientation as a mathematician and his orientation as a teacher. (This was noticed in studies by: Hannah, et al., 2011 and Paterson, et al., 2011.) For example, in one of our weekly discussions, he discussed how he was disappointed with a particular homework question on which they had spent significant time in class: calculating the degree of a field extension. He mentioned that, while most of the students seemed to understand the basic idea, they were sloppy in their execution. From a mathematician's standpoint, it is not only correctness that matters but also elegance. From a teacher's perspective, however, he resolved to be patient and decided to take the next class period to review this topic. Going on with new material, he believed, would have left a scar for the rest of the course (orientations). This compromise proved to be quite effective: the students seemed relieved. This example, in addition to providing information about the professor's orientations, also lays the groundwork for a more in-depth examination of the relationship between these two orientations as the course continues.

On another occasion the professor stressed the power of visualization of abstract concepts. This is in line with a study by Sfard (1994) in which she interviewed mathematicians in regards to the power of their visualization. In her view, "they stressed that pictures, whether mental or in the form of drawings, are only a part of the story. They support thinking, but they do not reflect it in all dimensions" (p. 48).

So far the effect of this collaboration has been positive in the sense that everyone in the group are not only focuses on the research mathematician's teaching strategies and thinking processes, but also their own teaching and decision making on day-to-day basis. Moreover, it has provided a platform allowing mathematicians to talk about mathematics freely and share their thought processes with each other.

We would like to ask the audience the following questions:

What are some themes that we could explore? What areas of study we should concentrate on that we haven't thought about? What future data we should plan to seek from? Is Nardi's "fragile" relationship between mathematicians and mathematics educators still an issue stopping the community to work closely together?

We hope to see more collaboration to produce tomorrow's mathematicians who are not only capable in doing high quality research, but also have a passion for teaching undergraduate courses.

References

Byers, W. (2007) How mathematicians think: Using ambiguity, contradiction, and paradox to create mathematics, Princeton University press.

Fischbein, E. (1994). The interaction between the formal, the algorithmic, and the intuitive components in a mathematical activity. In R. Biehler, R. W. Scholz, R. Straber, B.

Winkelmann (Eds.), Didactics of Mathematics as a Scientific Discipline (pp. 231-245). Dordrecht: Kluwer Academic Publishers. Printed in the Netherlands.
Hannah, J., Stewart, S., & Thomas, M. O. J. (2011). Analysing lecturer practice: the role of orientations and goals. International Journal of Mathematical Education in Science and Technology, 42(7), 975-984.
Hodgson, B. R. (2012). Whither the mathematics/didactics interconnection? Evolution and challenges of a kaleidoscopic relationship as seen from an ICME perspective. ICME conference, Plenary Presentation, Seoul, South Korea.
Mason, J. (2002). Mathematics teaching practice: A guide for university and college lecturers. Chichester, Albion/Horwood Publishing House.
Nardi, E. (2007). Amongst Mathematicians: Teaching and Learning Mathematics at University Level. New York; London: Springer.
Paterson, J., Thomas, M. O. J., & Taylor, S. (2011). Decisions, decisions, decisions: what determines the path taken in lectures? International Journal of Mathematical Education in Science and Technology, 42(7), 985-995.
Schoenfeld, A. H., How we think: A theory of goal-oriented decision making and its educational applications. New York: Routledge, 2011.
Sfard, A. (1994). Reification as the birth of metaphor. For the Learning of Mathematics, 14(1), 44-55.
Speer, N. M., Smith, J. P & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. Journal of Mathematical Behavior, 29, 99–114.
Thurston, W. P. (1994). On proof and progress in mathematics. BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, Volume 30, Number 2, pp. 161-177.